

Using the limit definition of the definite integral, and right endpoints, evaluate $\int_{-1}^3 (3x+1) dx$.

SCORE: ____ / 15 PTS

NOTE: Solutions using any other method will earn 0 points.

$$\begin{aligned} & \textcircled{1} \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(3 \left(-1 + \frac{4i}{n} \right) + 1 \right) \frac{4}{n} \textcircled{2} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left(-2 + \frac{12i}{n} \right) \textcircled{2} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \left(\sum_{i=1}^n -2 + \frac{12}{n} \sum_{i=1}^n i \right) \textcircled{1} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \left(\underbrace{-2n}_{\textcircled{1}} + \frac{12}{n} \cdot \frac{n(n+1)}{2} \right) \textcircled{1} \\ &= \lim_{n \rightarrow \infty} \left(-8 + \frac{24(n+1)}{n} \right) \textcircled{1} \\ &= -8 + 24 \textcircled{1} \\ &= 16 \textcircled{1} \end{aligned}$$

① FOR WRITING
 $\lim_{n \rightarrow \infty}$
ON EVERY LINE

A person's velocity as a function of time is shown in the graph on the right.

NOTE: The graph is a piecewise defined function whose "pieces" are all linear functions.

SCORE: ____ / 5 PTS

- [a] Estimate the distance the person travelled from time $t = 1$ second to $t = 9$ seconds using four subintervals and left endpoints.

$$\Delta t = \frac{9-1}{4} = 2$$

$$v(1)\Delta t + v(3)\Delta t + v(5)\Delta t + v(7)\Delta t$$

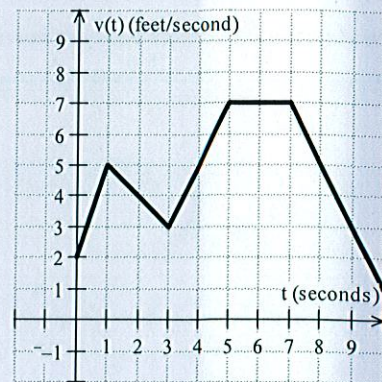
$$= \underline{5 \cdot 2} + \underline{3 \cdot 2} + \underline{7 \cdot 2} + \underline{7 \cdot 2} = \underline{44 \text{ ft}}$$

- [b] Find the exact distance the person travelled from time $t = 1$ second to $t = 9$ seconds.

$$\frac{1}{2}(5+3)(2) + \frac{1}{2}(3+7)(2) + 7(2) + \frac{1}{2}(7+3)(2)$$

$$= \underline{8} + \underline{10} + \underline{14} + \underline{10} \quad \left(\frac{1}{2}\right) \text{ POINT EACH}$$

$$= \underline{42 \text{ ft}}$$

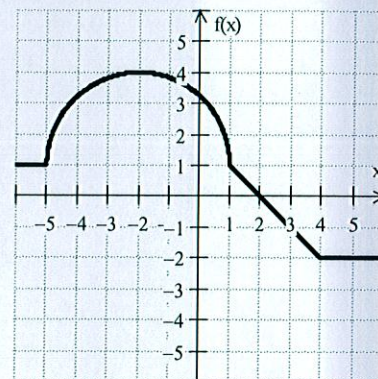


The graph of function f is shown on the right.

SCORE: ____ / 5 PTS

The graph consists of a horizontal line, a semi-circle with center $(-2, 1)$, a diagonal line and a horizontal line.

- [a] Evaluate $\int_{-5}^5 f(x) dx$.



- [b] Evaluate $\int_4^1 f(x) dx$.

$$= -\int_1^4 f(x) dx$$

① POINT EACH

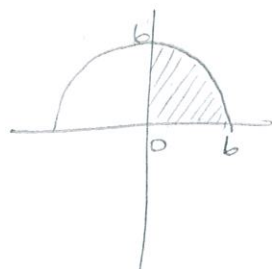
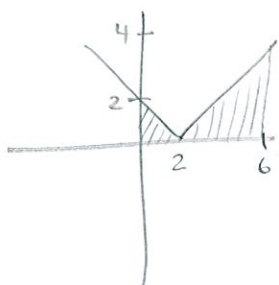
$$= -\left[\int_1^2 f(x) dx + \int_2^4 f(x) dx\right] = -\left[\frac{1}{2}(1)(1) + -\frac{1}{2}(2)(2)\right] = -\left[\frac{1}{2} - 2\right] = \underline{\underline{\frac{3}{2}}}$$

Evaluate $\int_0^6 (|x-2| + \sqrt{36-x^2}) dx$ using the properties of definite integrals

SCORE: ____ / 5 PTS

and interpreting in terms of area.

$$= \int_0^6 |x-2| dx + \int_0^6 \sqrt{36-x^2} dx = \frac{1}{2}(2 \times 2) + \frac{1}{2}(4 \times 4) + \frac{1}{4}\pi(6)^2$$



$$= \underline{2} + \underline{8} + \underline{9\pi}$$

$$= \underline{10 + 9\pi}$$

① POINT EACH